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THE DANGER OF STALLED FLIGHT AND AN ANALYSIS OF
THE FACTORS WHICH GOVERN IT

By

L. Hopf

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THE DANGER OF STALLED FLIGHT AND AN ANALYSIS OF
THE FACTORS WHICH GOVERN IT.*

By

L. Hopf

1. The cause of very many mishaps in aviation is so-called "stalled flight." By this is understood the phenomenon when, in steep climbing or in sharp curves, in short, in all kinds of maneuvers at low speed, a condition occurs in which the pilot loses control of his airplane. In less serious cases, a longer time and more room is required than in ordinary flying in order to recover control, but in the more serious cases the elevator or the rudder fails to work and the airplane falls. Usually an involuntary pitching flight takes place suggestive of the falling of a dry leaf. It is known that low speed is the essential cause of this dangerous condition and that it does not occur if the speed is not allowed to fall below a certain limit, which naturally differs for different airplanes. Hence, the oft-repeated instructions to carry an air-speed instrument on every flight and not, as was formerly customary, to trust alone to the r.p.m. for judging the speed. The condition of the engine alone does not assure the safety of flight.

2. It was early recognized that the phenomena in stalled flight were not connected with the pitching moments on the airplane and therefore not with what is termed static stability and

*Taken from Berichte und Abhandlungen der Wissenschaftlichen Gesellschaft für Luftfahrt, Sept., 1920, pp. 72-80.

instability, but with the peculiar relations of the lift at increasing angles of attack. The lift does not continue to increase indefinitely with the angle of attack (as the resistance does), but diminishes again after passing a certain maximum. This maximum angle of attack lies between 15° and 20° , not much different from that needed in ordinary flight. For the explanation of stalled flight, we come then to the following conclusions. If the angle of attack is increased, the resistance also increases. So long as the lift also increases with the increasing angle of attack, it can support the weight, but if the lift does not increase correspondingly, then the weight becomes greater than the lift and the airplane loses altitude. This conclusion can be stated exactly with the aid of Penaud power diagrams (Painleve, La Technique Aeronautique I, p.8).

For conditions of unaccelerated flight it is customary to plot the drag force, W , which must be overcome by the thrust of the propeller, against the velocity. The weight of the airplane is balanced by the lift if the angle of attack is properly adjusted. If the propeller thrust, S , is also plotted against the speed as in Fig. 1, the intersections of the two curves denote the limiting speeds in horizontal flight and the vertical distance between the curves is a measure of the climbing ability of the airplane. This method is so well known that no further explanation should be necessary. As we pass from the condition of equilibrium I along the W curve to higher speeds, the re-

quired resistance becomes greater than the actual propeller thrust and the speed must diminish until point I is again reached. In an analogous manner, it must increase, when the speed is lower than that corresponding to point I. Thus the condition of equilibrium I is shown to be stable and it may be shown in the same manner that the condition of equilibrium II for a lower speed is unstable. This process of reasoning does not apply to stalled flight. It would apply, if, in such a flight, the path of the airplane should always maintain the same inclination, especially if it should remain constantly horizontal. In no case, however, is this to be assumed. The conclusions drawn from Fig. 1 for the conditions of equilibrium are free from objections, as, for example, the determination of the steepest climb from the point where the difference between the ordinates of the two curves is greatest. It is therefore correct that, with large angles of attack belonging to lower speeds than v , an airplane can not climb so steeply as with the angle of attack belonging to v and that an airplane, which is forced by a strong pull into a condition of equilibrium with such an angle of attack, climbs less steeply. The so-called reversal of the steering effect here becomes noticeable. Such flights are not however at all dangerous and should not be designated as stalled flights. At greater heights the pilot will not perceive any way that he is flying at a lower speed, and consequently greater angle of attack, than corresponds to the steepest climb. There is

no question of lack of steering ability. Hence stalled flight and flight with reversed steering effect are to be sharply distinguished from each other. The fact will be considered below that the reversal of the steering effect, insofar as it does not depend on continuous effect, but only on the momentary influence of the elevator, is not included in the above exposition.

3. In order to understand the relations in stalled flight, we must therefore not only bear in mind the conditions of equilibrium and such static stability considerations, but also the accelerated flight and the disturbed equilibrium. There first presents itself the method of the customary dynamic considerations of stability, the method of small oscillations. I can also assume this method as known, since it was explained in detail at a recent session of the W.G.L. (Wissenschaftliche Gesellschaft für Luftfahrt) by Karman and Trefftz. I will only take up here the effect of these considerations on stalled flight. For the stability of longitudinal motions (that is, motions without curves) there are two quantities of decisive importance, both of which play a fundamental role in the balancing of rotation moments: the so-called static stability which depends on the position of the center of gravity and the static moment of the tail unit about the axis passing through the center of gravity, and the damping which opposes the pitching of the airplane and which depends chiefly on the moment of inertia of the tail unit about the center-of-gravity axis. Quittner and Karman and Trefftz

have shown that without static stability an airplane can not have dynamic stability and that even with static stability an airplane may be dynamically unstable, if the damping is not great enough. Said authors calculated numerically only the relations for a practicable angle of attack, which may be suitable for the steepest flight, and represented the result on a diagram in which the static stability is shown by the ordinates and the damping by the abscissas. In Fig. 2, G stands for weight, J for moment of inertia, F for wing surface of airplane, M for torsion moment of the air forces on the whole airplane, M_H for torsion moment on the tail unit alone, γ for the specific weight of the air, α for the angle of attack, q for dynamic pressure, v_H the distance of the tail surface from the center of gravity of the airplane. Curve I separates the stable from the unstable field. Airplanes, whose static stability and damping fall in the field inclosed by Curve I and the ordinate are unstable in spite of positive static stability, while the other values of the drawn quadrants give stable airplanes. Negative damping has no meaning. Negative static stability leads constantly to instability. It is now of interest for our problem, as to how Curve I is changed, if we take other angles of attack, especially those in the vicinity of the maximum lift, as the basis of our calculation. Curve I bulges out with increasing angle of attack. For a value still lying below the maximum lift (but whose more accurate expression here would lead us too far), it contains an

infinitely distant point, that is, there is a value of positive static stability, at which no damping suffices for dynamic stability. At a somewhat higher value, the lower branch of this curve coincides with the abscissa, so that with a smaller or only moderate static stability no dynamic stability is longer possible and only with greater static stability is there another field of dynamic stability. Thereby it is of interest that the static stability must increase with the damping. So far as I can judge the numerical values, they would seem moreover to give stability in this field for a normal airplane. In these calculations there remains much that is physically unsatisfactory. The determination of the stability or instability still gives no explanation of the actual processes. Even if instability could be determined numerically for a large angle of attack, it would not make clear the dangers of stalled flight. If a condition of equilibrium is unstable, it will not be actually attained. It will only be possible to maintain a sort of balance with the aid of steering devices in its vicinity. Toward every motion of the elevator, the condition of the airplane in this position will be especially sensitive, not insensitive, as experience with the condition of stalled flight teaches. Under some circumstances, an unstable airplane will fly quite well and safely. Such instability is surely of quite a different sort from the dangerous condition which occurs in stalled flight. In order to arrive at a clear conception of this difference, we must follow an airplane in its

accelerated motion on a disturbed path. We must not confine ourselves to infinitely small oscillations (in the immediate vicinity of equilibrium conditions), but we must consider the course of the whole longitudinal motion of an airplane.

4. This consideration demands the integration of three motion formulas of flight without lateral motion. The problem is generally very difficult mathematically, especially when the air powers are given only in empirical, not analytical, dependence on the angle of attack. The integration is however successful, in a satisfactory and comprehensive manner for all practical needs, through the knowledge that the great forces perpendicular to the path come into equilibrium considerably quicker than the relatively small forces operating in the direction of the path. Professor Fuchs and I explained this thoroughly in the last number of the "Technische Berichte" (R. Fuchs and L. Hopf, "Die allgemeine Längsbewegung des Flugzeugs," 1st part. T.B. III, p. 317) and gave a method of calculation, which however may still make a complicated impression on the impartial reader. We will not make calculations here, but only explain the physical relations. We will nevertheless also keep in mind the differential equations.

The equilibrium of the forces in the flight direction requires: mass $\times$ acceleration = propeller thrust - weight component - head resistance, with the familiar relations:

$$\frac{G}{g} \frac{dv}{dt} = S - G \sin \varphi - c_w \frac{\gamma}{2g} v^2 F \quad \dots (1)$$

The equilibrium of the forces perpendicular to the flight direction requires: centrifugal force = lift-weight component.

$$\frac{G}{g} v \frac{d\varphi}{dt} = c_a \frac{\gamma}{2g} v^2 F - G \cos \varphi \dots \dots \dots (2)$$

It must here be expressly emphasized that φ stands for the angle of flight with the horizontal. The changing of the angle φ , with the consequent curving of the path of flight, is the result of disturbing the forces perpendicular to the path. The angle θ (Fig. 3) formed by the airplane axis and the horizontal, is composed of the climbing angle φ and the angle of attack α added together.

$$\theta = \varphi + \alpha \dots \dots \dots (3)$$

This angle θ gives the position of the airplane in space. It plays no role in the equilibrium of forces for which the airplane is only a material point. It is the deciding factor in balancing the rotation moments, which are entirely independent of the direction of gravity and consequently of the angle φ .

The equilibrium of the moments demands moment of inertia $\times$ torsion acceleration = - moment of head resistance due to elevator position s and angle of attack α - damping moment.

$$J \times \frac{d^2\theta}{dt^2} = - m (s, \alpha) v^2 - n \frac{d\theta}{dt} v \dots \dots \dots (4)$$

The flight path bends under the influence of the forces perpendicular to it, the airplane pitches under the influence of the moments, and the reaction of the air on the airplane in connec-

tion with both these motions depends essentially on the angle of attack, that is, on the position of the airplane with reference to the course followed. Only by changing the angle of attack does the force affect the position of the airplane in space and the moment affect the flight path. These relations impart to the disturbed and guided motion of the airplane its special character. The speed changes only under the influence of the relatively small forces occurring in equation (1) and this change (when one refrains from direct interference with this equilibrium by starting or stopping the engine) is far smaller and slower than the change of the three angles φ , θ and α . Only when the vertical forces are balanced, when consequently the curvature of the path is small, can the speed acceleration or diminution play a decisive roll. Professor Fuchs amply demonstrated this with examples in the above-mentioned T.B. article.

5. If the motion of the airplane is represented in a field of the coordinates v , α , θ , the flight conditions, under which the vertical forces are in equilibrium, will be represented by a plane surface. The intersections of this plane with different levels $\theta = \text{const.}$ are shown in Fig. 4. It is apparent how small the dependence of this curve is on θ in the realm of all climbs and horizontal glides ($20^\circ > \theta > -20^\circ$). Hence the level representation of the motion in a (v, α) system of coordinates is the most practical. Any point of the (v, θ, α) space can, through any kind of disturbance, represent the initial condition of an accelerated motion. This motion will always be of such a

character that the initial speed will remain almost unchanged and the point representing the airplane moves in a plane perpendicular to the v -axis of the equilibrium plane of the vertical forces. In the latter plane, according to equation (1) $\frac{d\varphi}{dt} = 0$, the path will not be bent. In space I, $\frac{d\varphi}{dt} < 0$ and the path will accordingly be bent downward. In space II, $\frac{d\varphi}{dt} > 0$ and the path will be bent upward. If the speed falls below the value requisite for equilibrium at the given angle of attack, the path curves downward, the climb becomes flatter, and the airplane shows a tendency to sink. In passing beyond the plane where $\frac{d\varphi}{dt} = 0$, this tendency ceases, the path curves upward and indeed proportionally to the increase in the angle of attack.

But if, through some disturbance, the speed becomes so slow, that the point in this motion in the (v, θ, α) space does not hit the plane of equilibrium of the vertical forces, then the whole motion of the airplane maintains its initial direction. The path constantly curves further downward and the airplane has a constantly increasing tendency to fall. This is now the case of the stalled flight. Essential for its inception is the falling of the speed below the minimum value at which the equilibrium of the vertical forces is possible. It does not matter at what angle of attack this occurs.

6. We have, in accord with experience, emphasized the essential conditions conducive to stalled flight, but the above conclusions refer only to the path of flight, not yet to the position

of the airplane, and consequently to the quantity θ , upon which the angle of attack and the air forces essentially depend. For this purpose we must go more minutely into the equilibrium of moments. The equilibrium of moments which, according to equation (4) is given by

$$m(s, \alpha) = 0 \quad (5)$$

determines the angle of attack as determined by the action of the elevator s . If the equilibrium is so disturbed that the angle of attack deviates from its equilibrium value, a moment is created, which, with static stability and increasing angle of attack, presses the airplane down stronger in front, or vice versa in case of static instability. These well known relations do not need further consideration here, as Fig. 5 makes them plain. Top-heavy working moments are thereby calculated as positive.

In order to consider the simplest case of stalled flight, we will assume that the airplane speed has been lowered by some disturbance to the necessary small value, but that the angle of attack has retained its equilibrium value. Then no moment is at first created and the position of the airplane in space does not change, but the flight path curves downward. The angle of attack consequently increases. By this increase a secondary moment is generated which turns the stable airplane downward toward the flight direction, but turns the unstable airplane upward away from the flight path. For a stable airplane, therefore, the angle of attack increases slower than for an unstable one. A neutral air-

plane is not turned at all by such a disturbance. The turning downward of an airplane causes a constant increase in the angle of attack. The course of such a motion is shown in the said T.B. article. For large angles of attack, entirely unexplored conditions of motion enter in. If, however, the airplane is statically stable, the airplane axis will be drawn more strongly toward the path of flight, as the attacking angle increases, and consequently a limit is soon set to this increase. Fortunately, within the range of the attacking angles which belong to maximum lift, all air planes are very stable, since the turning moment of the air forces on the wings varies only slightly. The center of pressure remains at the same place when the angle of attack increases, and the value of the lift coefficient also varies but slightly. The stabilizing influence of the tail unit, which, under normal flight conditions, is partially or entirely eliminated, has its full effect in stalled flight.

7. We do not need therefore to place a very high value on the danger of the attacking angle's automatically exceeding all reasonable limits. Perhaps the present general safety of flight could not have been attained if the wing moments had not had this property. The actual danger in stalled flight lies only in the course of the flight path itself.

In the quantitative calculation with reference to the course of the moments, we here find again the above qualitatively determined behavior. The flight path sinks and curves constantly

further downward, without its being possible by pulling to do anything to prevent it. This is shown by the following comparison of two different flight conditions of the same airplane.

Let the weight G of the airplane be 1530 kg. and its wing surface $F = 41.3$ sq.m. Let it be located at such an altitude that the air density $= 0.108 \left(\frac{\text{kg}}{\text{m}^3} \right)$. Let the propeller thrust S (in the realm of flight speeds between 22 and 26 m/sec. adapted to our example) be approximately represented by the equation

$$S = 400 - 0.110 v^2 \quad (3)$$

which, for 170 HP, corresponds to an efficiency of about 60%.

The coefficients of lift and drag, in dependence on the angle of attack are contained in Fig. 6. From these data is obtained the determining value $\frac{d\varphi}{dt}$, shown in Fig. 7, for the bending of the flight path, dependent on the angle of attack and on the speed.

Let the radius of gyration of the airplane be 1.4 m., so that the moment of inertia $J = 310 \text{ (kg m s}^2\text{)}$ and the distance u_H of the center of gravity of the tail unit from the center of gravity of the airplane be 5.7 m. In Fig. 8, the moments M_H of the tail-unit forces and the moment of the total air forces (with reference to the airplane), is laid out on the unit of dynamic pressure q , the latter for the cases where, in the realm of normal angles of attack, either static stability, neutrality, or instability occurs. Such a position of the elevator is assumed, that equilibrium exists when $\alpha = 8^\circ$. In our example the calculation is carried out for this position and for the equilib-

rium condition $\alpha = 12^\circ$. In the latter case, the curve must be bent down so far that it cuts the abscissa at $\alpha = 12^\circ$. The effect of the elevator is shown by simply changing the curve of the moments upward for pushing, downward for pulling. In the following example a displacement of $\Delta \frac{M}{q} = \pm 1.5$ cu.m. is assumed for push or pull. The quantities occurring in equation (4) are expressed in the following manner:

$$m = \frac{\gamma}{2g} \frac{M}{q} \dots \dots \dots (7)$$

$$n = v_H \frac{\gamma}{2g} \frac{\partial}{\partial \alpha} \frac{M_H}{q} \dots \dots \dots (8)$$

In our example $n = 0.0335$ (kg s²).

The calculation is easily made. We proceed from a given starting position and, for a definite interval, subtract from the angle of attack in Figs. 7 and 8 the average values of the quantities there represented. Equation (4) is then a linear differential equation, from which we take away the value $\frac{d\theta}{dt}$

$$\frac{d\theta}{dt} = -\frac{m}{n} v - \left(-\frac{m}{n} v - \left[\frac{d\theta}{dt} \right]_0 \right) e^{-nv/J} \dots \dots \dots (9)$$

Hereby $\left(\frac{d\theta}{dt} \right)_0$ indicates the initial value in the interval under consideration. From $\frac{d\theta}{dt}$ and $\frac{d\varphi}{dt}$ follows however $\frac{d\alpha}{dt}$; with the help of these three values, we proceed gradually and easily with the further calculation. In larger intervals we calculate according to equation (1) the increment or decrement of the speed and make allowance for this change in the use of Fig. 7. Also in

solving $\frac{d\theta}{dt}$, allowance is readily made in the calculation for the change of speed. Still it plays no great role there, as long as the speed variations are small.

8. Figs. 9 and 10 show the course of the three angles α , φ , and θ in two cases, which clearly show the contrast between normal and stalled flight. The disturbance is the same in both cases. The speed has fallen 3.45 m/s below the value required by the angle of attack and by the climbing angle. The equilibrium conditions are:

Case A	$\alpha = 8.0^\circ$	$\varphi = 4.8^\circ$	$\theta = 12.8^\circ$	$v = 28.2 \text{ m/s}$
Case B	$\alpha = 12.0^\circ$	$\varphi = 4.1^\circ$	$\theta = 16.1^\circ$	$v = 25.2 \text{ m/s}$

Case A deals with the equilibrium condition corresponding to the steepest climb. In Case B, the climbing angle is not much less. The maximum of φ runs very flat. It may hardly be assumed that the pilot can feel whether he is in the one or the other condition. The angle θ between the horizontal and the airplane axis is in Case B considerably larger than in Case A, but it can be concluded only from the barogram, that the climb is less steep.

When, through some disturbance, the speed in both cases falls 3.45 m/s, the first effect will be a downward curving of the path, since the airplane is then in field 1 of Fig. 4. From Fig. 7 it is evident that the curvature in Case B is much greater. Therefore the stability is greater in Case B and the airplane is turned quicker toward the flight path than in Case A. If the elevator is not brought into play, the path quickly becomes flatter

in Case A, but even after two seconds the curvature has become quite small, while the flight path continues to climb. In Case B, the climbing angle becomes 0 in not quite one second, and the flight path continues to curve downward, when the airplane settles and begins to fall. If it is not high above the earth, there is great danger. At great heights the danger is not great and with the lapse of time the flying course is automatically resumed as the speed increases. In aerial combat even a short fall may be decisive.

9. The real character of stalled flight comes sharply into view when we follow mathematically the result of changing the position of the elevator in both cases. When the flight path curves downward, there is a natural impulse to pull, that is, to right the airplane. The elevator is pulled so that a downward force is exerted upon it which raises the nose of the airplane. The pitching of the airplane affects the angle of attack and thus immediately the course of the airplane. The quantitative relations may be gathered from the figures. In both cases the angles α and θ are enlarged by pulling, but the angle φ , upon which the flight course alone depends, varies.

In Case A, the curvature becomes zero in $3/4$ second. It then curves upward and climbs steeper and steeper. By the end of two seconds it reaches the original value of the climbing angle, which it will then considerably exceed. If we recall that the initial condition in Case A corresponds to the steepest con-

tinuous climb, it is then evident from this example how easily the above comparison of equilibrium conditions may lead to false conclusions. The conclusions there drawn correspond to permanent conditions consequently only to the effect of a change in the position of the elevator after the lapse of considerable time, until a condition of equilibrium has become established. Their transfer however to momentary conditions is false. For the momentary effect of moving the elevator, which is the question in most instances, there is no question of a reversal of the effect of the elevator. If the airplane, through pulling on the elevator, is removed from the equilibrium condition of the steepest climb, or a somewhat higher angle of attack, then the flight path goes still steeper than the angle corresponding to the condition of equilibrium. The effect is just what is expected from the natural feeling.

The elevator produces quite a different effect in Case B. Here the behavior of φ in pulling varies so little from its behavior when no impulse is given the elevator that both curves in Fig. 10 fully coincide. The airplane is rightly turned by the elevator, but the flight path does not go with it. The diminution of $\frac{d\varphi}{dt}$, corresponding to the increase in the angle of attack according to Fig. 7, is in this case fully offset by the smaller climbing of the speed in pulling. If the pilot allows himself, through the failure of a light pull, to be influenced to give a stronger pull, then the angle of attack becomes still greater and, according to Fig. 7, the flight path curves still more down-

ward. The speed will no longer increase and finally it will sink again. Then the catastrophe is unavoidable. Fig. 11 shows the speed variations in all the calculated cases.

Alongside the effect of pulling, there is shown in Figs. 9 to 11 the effect of pushing. In Case A this is of no further interest. In Case B it consists, on the one hand, of a greater downward curving of the flight path and, on the other hand, of a correspondingly rapid increase of speed. After only two seconds, the latter is so great that the character of the disturbance approaches Case A, so that consequently through further pulling the flight path can be quickly righted. "First push, then pull" must be the instructions for a pilot who desires to get out of the stalled flight condition. The righting of the flight path, in the case of stalled flight, is not to be attained through the quick balancing of the vertical forces as in Case A, but only through the comparatively slow balancing of the forces in the direction of flight. Contrary to the first natural impulse, this balancing can be greatly hastened by pushing.

10. Our discussion puts us now in a position to judge as to what external conditions and what structural measures influence the inception and the dangers of stalled flight in a favorable or unfavorable manner. First of all, it is clear that disturbances of the said kind are harmless, when it is possible to initiate immediately the balancing of the forces acting in the direction of flight and when accordingly the speed can be instant-

ly increased through increased engine power. This is especially true if the disturbance occurs in gliding flight when the whole engine power is available as reserve power. Also in the case of great reserve power on the ground (hence on airplane with great climbing ability) the danger is diminished, since they climb very steeply near the ground with a large attacking angle. The initial value of φ is consequently large and hence does not become negative so soon. It is longer before the flight path sinks.

In gliding flight the stalled flight condition is therefore less likely to occur, since the deciding curve on the (v, α) plane (Fig. 4) applies to smaller speeds (with negative θ) than in climbing. It is true moreover that a smaller angle of attack can be used in gliding than in climbing. The attacking angle of the flattest glide is smaller than the attacking angle of the steepest climb.

11. The construction of the airplane may exert an influence on stalled flight in two ways: First, there are measures which hinder the inception of stalled flight, and secondly, there are measures which facilitate emerging from the same. The inception becomes more difficult the further the minimum speed of normal flight differs from the speed at which equilibrium of the vertical forces is no longer possible. The difference between the two speed values is however proportional to the difference $\delta \sqrt{c_a \frac{F}{G}}$, and is consequently, for the given surface loading, influenced

by the difference in the two lift values. This difference depends very largely on the wing section. We can differentiate two types (Spad and Fokker in Fig. 12): One, with small drag and lift coefficients, is preferred for swift climbing airplanes; the other, with large coefficients, for good climbing airplanes. The value of the maximum lift lies, for the second one, farther from the lift value for the steepest climbing than for the first one. Experienced test pilots have called attention to the fact that the Spad is more easily stalled than the Fokker. Measurements in the Göttingen tunnel gave for the two sections.

$$\text{Spad} \quad DD \quad \delta c_a = 0.23$$

$$\text{Fokker} \quad DD \quad \delta c_a = 0.37$$

It was nevertheless evident that the measurement of the Fokker section in the field of large attacking angles was very uncertain and gave widely differing results. That such an uncertain state also occurs in the dimensions of an actual airplane seems improbable, according to the statements of the pilots. One would have to expect great uncertainty in the region of large attacking angles, while the contrary is according to experience.

The maximum lift diminishes and the lift component of the steepest climb increases relatively to it, when the induced resistance becomes greater. The danger of stalled flight must consequently increase with poor secondary relations or unfavorable arrangement of wings on multiplanes. This is confirmed by the statements of an airplane pilot, according to which the Fokker

DVII is stalled with difficulty, but, on the contrary, the Fokker triplane, which must have a very great induced resistance, is easily stalled. The wing sections of the two airplanes are alike. A great structural resistance must likewise facilitate stalled flight. The maximum lift is indeed not affected by such resistance, but the lift coefficient of normal ascent becomes greater, as is readily shown by polar diagrams (Fig. 13). Aside from the polar diagram, the inception of stalled flight is affected only by the surface loading and indeed the deciding difference in speed becomes greater with increasing surface loading. Heavy surface loading therefore has a favorable influence in this respect.

12. In stalled flight all influences must be regarded as favorable which hinder the pitching of the flight path and consequently lower the value of $\frac{d\varphi}{dt}$, and likewise all influences which facilitate the pitching of the airplane, especially by pushing and consequently raise the value of $\frac{d\theta}{dt}$.

If we designate by δv the loss of speed on account of the disturbance, we may then give equation (2) the form:

$$v \frac{d\varphi}{dt} = c_a \gamma \frac{F}{G} v \delta v \dots \dots \dots (10)$$

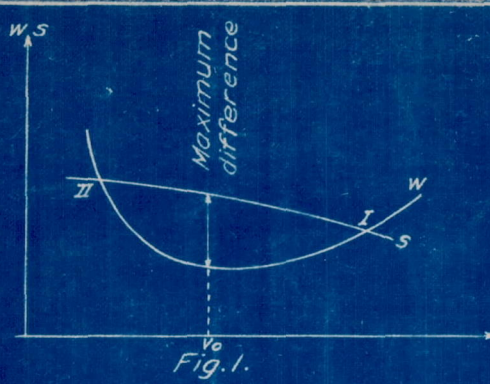
The absolute value of the speed does not therefore influence the change in inclination of the flight path $\frac{d\varphi}{dt}$, but the speed diminution is greater for swift airplanes. Here also large surface loading is favorable, since (other things being equal) the speed

is proportional to $\sqrt{\frac{G}{F}}$, so that on the right side (10) the factor $\sqrt{\frac{F}{G}}$ always remains and consequently the speed diminishes more gradually for large surface loading.

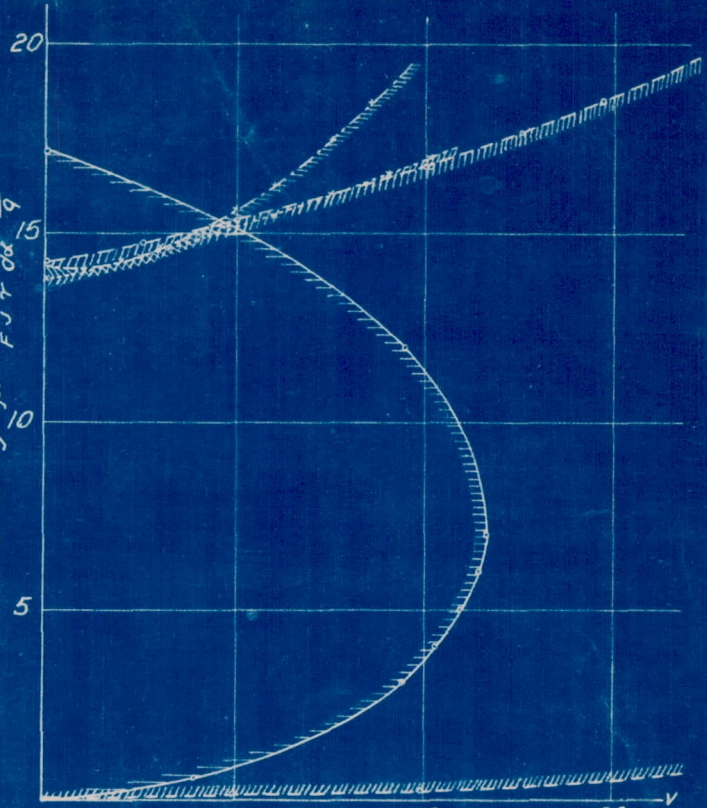
Fig. 8 and equation 9 show the different influence on the pitching of the airplane. Static stability facilitates (in connection with the here deciding disturbances) the effect of pushing, while static instability facilitates the effect of pulling. Consequently static stability is an advantage. Still, this advantage is not very important for the construction, since in this particular field of stalled flight all airplanes are stable. The size of the tail unit has no influence on the quantity $\frac{m}{n}$ in equation (9), which is the most important for pitching the airplane, but the length of the fuselage l_H , by which the damping is determined, appears in the denominator. The greater l_H , the greater the damping and the smaller the pitching speed. Consequently a great length of fuselage exerts an unfavorable influence. Aside from aerodynamic values, $\frac{m}{n} v$ is proportional to $\frac{V}{V_H}$. Hence large slow airplanes are less favorable in this respect than small swift ones. The influence of the moment of momentum comes into play only in the exponent of the e function in equation (9). The pitching is hindered more by a large moment of inertia. The insensibility of the flight path to elevator impulses is connected with the fact that in every disturbance of the kind we have considered, the angle of attack passes quickly into the field where $\frac{dc_a}{d\alpha}$ is very small. Only the airplane is

pitched by the elevator and thus the attacking angle is affected: First, through the dependence of the angle φ on α is an effect on the flight path possible. The quantity $\frac{d\varphi}{d\alpha}$ is however proportional to $\frac{dc_a}{d\alpha} \times \frac{F}{G}$. The influence of the elevator on the flight path is therefore small for a small value of $\frac{dc_a}{d\alpha}$ and indeed just so much smaller, the larger the surface loading is. Here we may establish an unfavorable influence of heavy surface loading. Whether this or the just mentioned favorable influence has the greater significance, can not be decided by the theorist. The unbiased opinions of experienced aviators or, still better, measurements of speeds and angles in stalled flight are required in connection with theoretical considerations in order to make them fruitful.

Translated by the National Advisory Committee for Aeronautics.

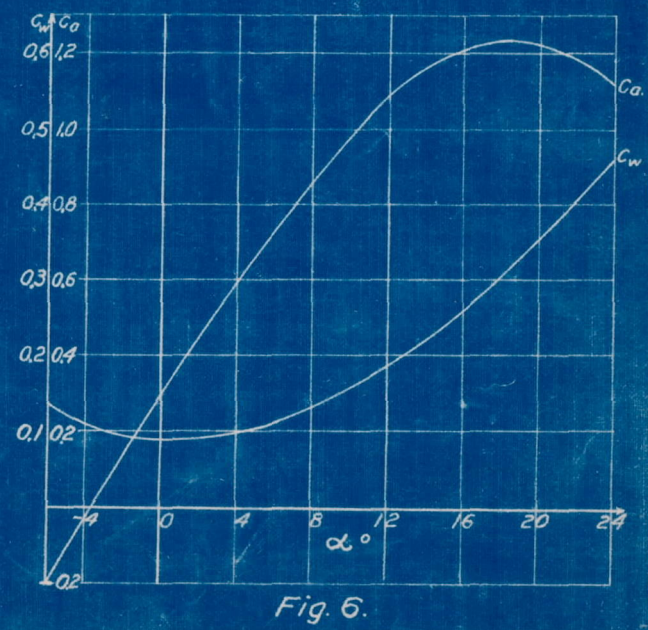
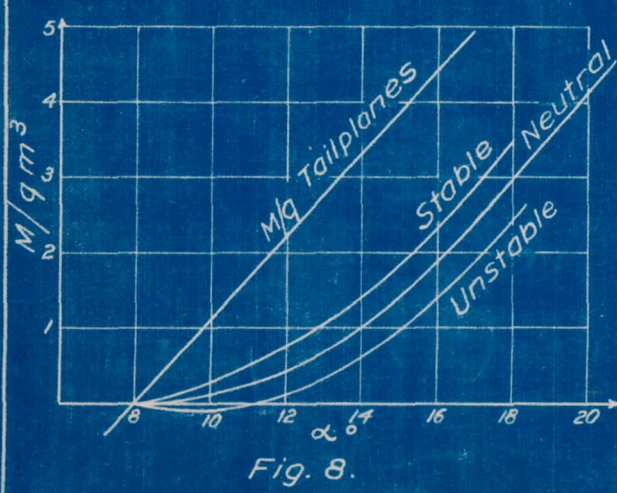
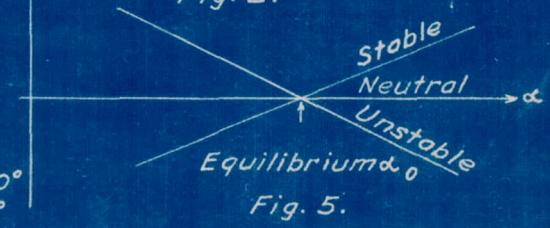
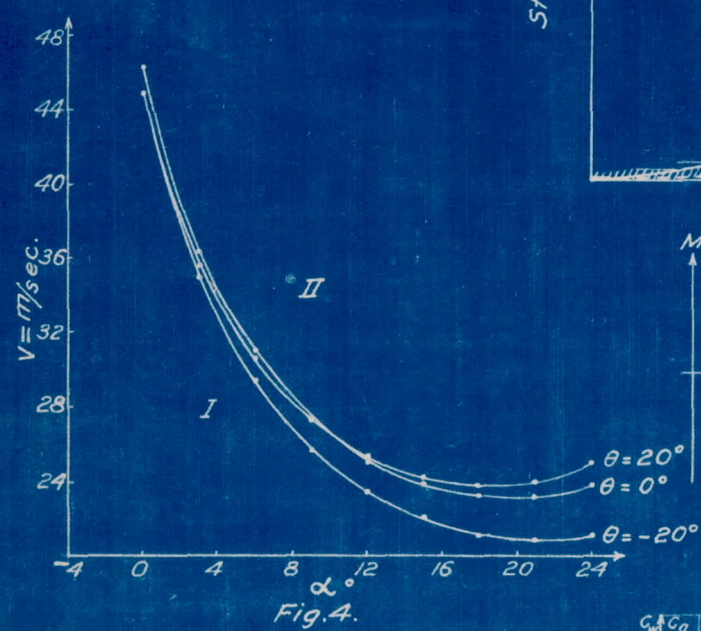


$$\text{Static stability} = \mu = \frac{2G^2}{FJ} \frac{\partial}{\partial \alpha} \frac{M}{q}$$



$$\text{Damping} = v = \frac{20}{JF} \frac{\partial}{\partial \alpha} \frac{M_H}{q}$$

Fig. 2.



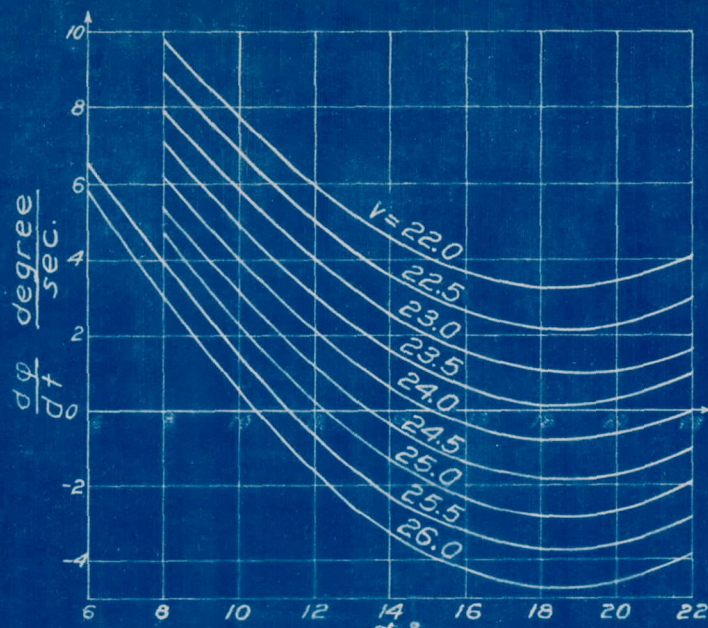


Fig. 7.

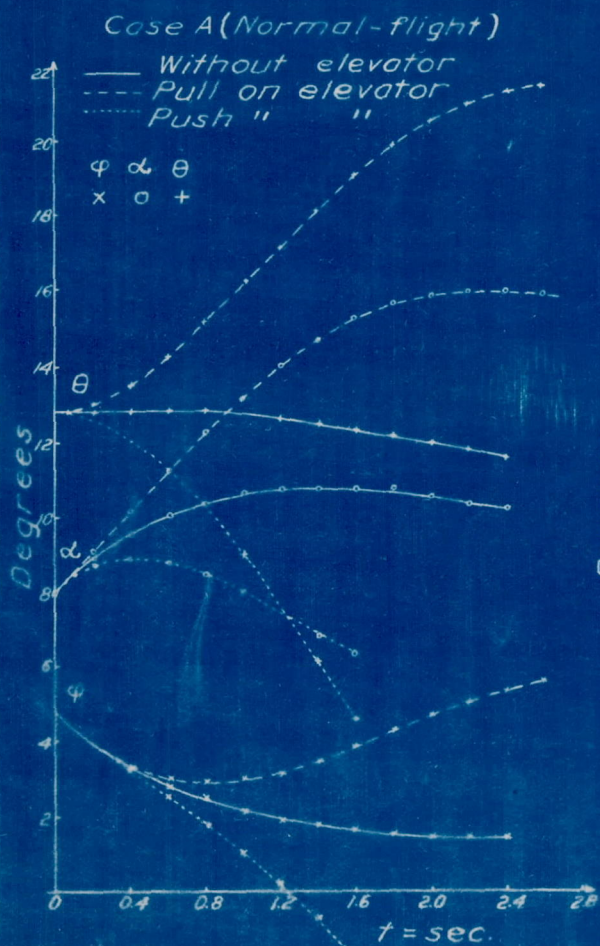


Fig. 9.

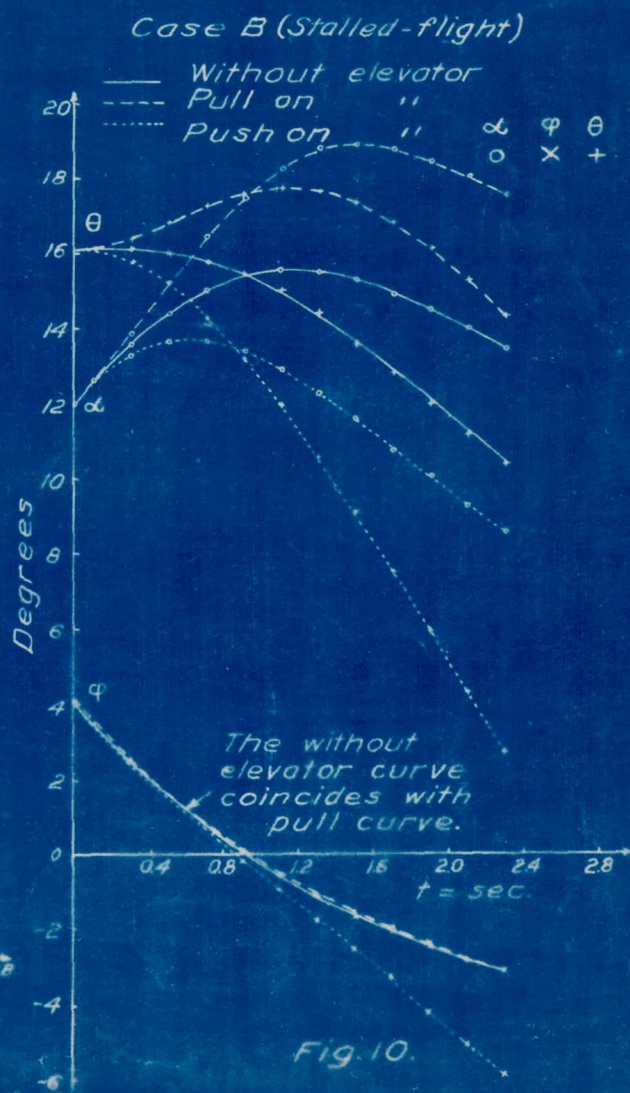


Fig. 10.

